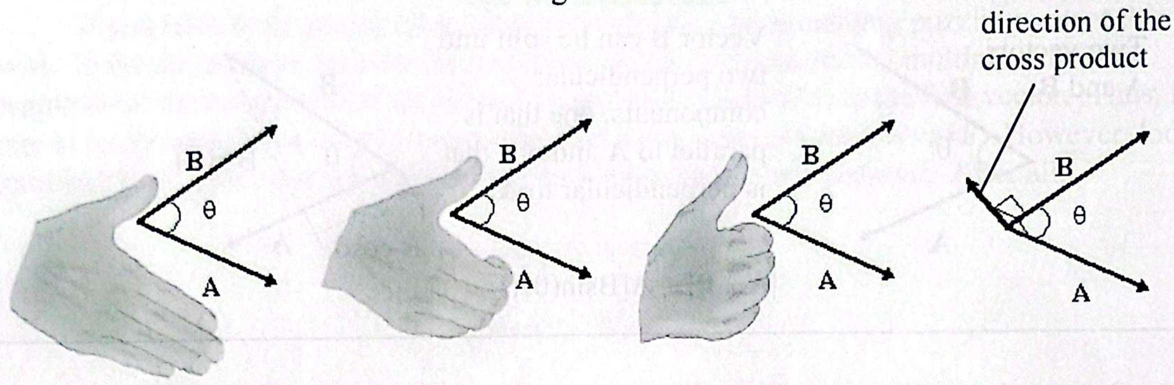
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FIGURE 1.9

The Right hand Rule



Point the fingers of your right hand in the direction of the first vector.

Curl your fingers along the arc of the angle between the vectors.

Your thumb points in the direction of the cross product.

In this example, the thumb points up out of the page. That's the direction of the cross product.

When determining the direction of the cross product, then, you take your right hand and point your fingers in the direction of the first vector. Then, you curl your fingers towards the second vector, along the arc of the angle between them. Your thumb will then point in the direction of the cross product. Think about what this means for a moment.

The direction of the cross product of two vectors will <u>always</u> be perpendicular to *both* vectors.

The right hand rule has another implication. Look at Figure 1.9 and use the right hand rule to determine the direction of **B** x **A**. Remember, you point the fingers on your right hand in the direction of the *first vector* (**B**), and you then curl towards the *second vector* (**A**). Where does your thumb point? It points down towards the paper. That's the *opposite direction* as that shown in the figure. Thus, unlike the dot production, the cross product is *not commutative*.

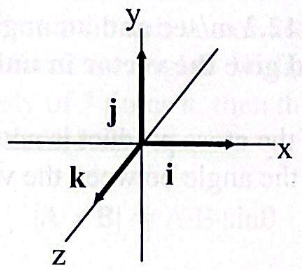
The cross product is not commutative: A x B = -(B x A)

This is important to remember.

Okay, there is one more thing you need to learn about the cross product. You need to know how to calculate the cross product using unit vector notation. However, before you can do that, you need to see how we define unit vectors in three-dimensional space.

FIGURE 1.10

Three-Dimensional Space in Terms of Unit Vectors



When we add another dimension (the z-axis), we can represent that in unit vector notation with just another unit vector. Notice that **i** is still the unit vector in the horizontal direction and **j** is still the unit vector in the vertical direction. To represent three-dimensional space, then, we simply add a third unit vector, **k**, which points out of the plane of the paper, towards you. If we multiply by -1, the unit vector -**k** points behind the paper.

Now that you know how three-dimensional space is represented in vector notation, you can learn how we compute the cross product. First, let's start with the simple case of two-dimensional vectors.

For two-dimensional vectors in the i/j plane: $\mathbf{A} \times \mathbf{B} = (A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k}$ (1.10)

Notice the restriction placed on this equation. To use this equation, you must be taking the cross product of two-dimensional vectors which exist only in the plane defined by the horizontal (**i**) and vertical (**j**) axes. Thus, this is a pretty restrictive equation. However, most of the physics problems that you do will involve such vectors, so it is probably what you will use most often for calculating cross products.

For completeness sake, I will give you the total equation for calculating the cross product between any three-dimensional vectors:

$$\mathbf{A} \times \mathbf{B} = (A_y \cdot B_z - A_z \cdot B_y) \cdot \mathbf{i} + (A_z \cdot B_x - A_x \cdot B_z) \cdot \mathbf{j} + (A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k} \quad (1.11)$$

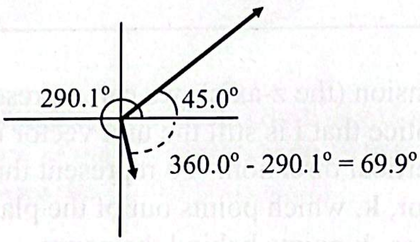
Notice that Equation (1.11) reduces to Equation (1.10) for two-dimensional vectors in the **i**/**j** plane. After all, the z-component of such a two-dimensional vector is zero. Thus, the term $(A_y \cdot B_z - A_z \cdot B_y) \cdot \mathbf{i}$ is zero, as is the term $(A_z \cdot B_x - A_x \cdot B_z) \cdot \mathbf{j}$. As a result, the only term in the equation that is non-zero is $(A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k}$, and that gives us Equation (1.10).

I know that this is a lot to throw at you, but hopefully the following example problems will clear up any confusion that may still be in your mind.

EXAMPLE 1.10

A velocity vector has a magnitude of 56.1 m/sec and an angle of 45.0 degrees. Another velocity vector has a magnitude of 12.2 m/sec and an angle of 290.1 degrees. Calculate the magnitude of the cross product and give the vector in unit vector notation.

Calculating the magnitude of the cross product is not bad at all. We just use Equation (1.9). To do that, however, we need the angle between the vectors. To determine that we will have to draw the two vectors:



If the angle of the second vector as defined from the positive x-axis is 290.1, then the angle from the positive x-axis *down* to the vector must be 69.9°, because the total angle must be 360.0°. Well, the angle from the positive x-axis *up* to the first vector is 45.0°. The angle between the two vectors, then, must be 69.9° + 45.0° = 114.9°. Now we can use Equation (1.9):

$$|\mathbf{A} \times \mathbf{B}| = A \cdot B \cdot \sin\theta = (56.1 \text{ m/sec}) \cdot (12.2 \text{ m/sec}) \cdot \sin(114.9°) = 621 \text{ m}^2/\text{sec}^2$$

The magnitude of the cross product, then, is <u>621 m^2/sec^2</u>.

What about the direction? For that, we use the right hand rule. We point the fingers of our right hand in the direction of the first vector, then we curl along the arc of the 114.9° angle in between the vectors. When we do that, our thumb points down into the page. Thus, the vector is in the negative **k** direction. That tells us what we need to know for unit vector notation. After all, we know the vector's magnitude (621 m^2/sec^2), and we know that it is pointed in the negative **k** direction. Thus, the vector is <u>$-(621 \text{ m}^2/\text{sec}^2) \cdot \mathbf{k}$</u>.

Given the following vectors, calculate A x B and determine the angle between the vectors:

$$\mathbf{A} = 3.4 \cdot \mathbf{i} + 4.5 \cdot \mathbf{j}$$
$$\mathbf{B} = 2.4 \cdot \mathbf{i} + 1.1 \cdot \mathbf{j}$$

Notice that these two vectors are two-dimensional and have only **i** and **j** unit vectors. Thus, they are in the **i**/**j** plane, and we can use the simpler version of the cross product equation, Equation (1.10):

$$\mathbf{A} \times \mathbf{B} = (A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = [3.4 \cdot 1.1 - 4.5 \cdot (2.4)] \cdot \mathbf{k} = \underline{-7 \cdot \mathbf{k}}$$

That's the cross product. Based solely on the unit vector notation, you know that it is pointing behind the plane of the paper. What about the angle? Well, we now know the magnitude of the vector (if the vector is composed solely of 7 times **k**, then the magnitude is 7, because **k** has a magnitude of 1), so we can use Equation (1.9):

$$|\mathbf{A} \times \mathbf{B}| = A \cdot B \cdot \sin\theta$$

Remember, "|**A** x **B**|" means "magnitude of **A** x **B**," so that's 7. We don't have the magnitudes of **A** and **B**, but we can use their components to calculate them:

$$A = \sqrt{3.4^2 + 4.5^2} = 5.6$$

$$B = \sqrt{(2.4)^2 + 1.1^2} = 2.6$$

Now we can use Equation (1.9):

$$|AxB| = A \cdot B \cdot \sin\theta$$

$$7 = (5.6) \cdot (2.6) \cdot \sin\theta$$

$$\theta = \sin^{-1}\left(\frac{7}{(5.6) \cdot (2.6)}\right) = \underline{30^\circ}$$

Given the following vectors, calculate the cross product and the angle between them:

$$\mathbf{A} = -1.4 \cdot \mathbf{i} + 4.2 \cdot \mathbf{j} - 5.6 \cdot \mathbf{k}$$
$$\mathbf{B} = 2.4 \cdot \mathbf{i} - 1.1 \cdot \mathbf{j} + 3.2 \cdot \mathbf{k}$$

This is essentially the same as the problem above. However, these are three-dimensional vectors, so we have to use the larger formula, Equation (1.11):

$$\mathbf{A} \times \mathbf{B} = (A_y \cdot B_z - A_z \cdot B_y) \cdot \mathbf{i} + (A_z \cdot B_x - A_x \cdot B_z) \cdot \mathbf{j} + (A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = (4.2 \cdot 3.2 - [-5.6] \cdot [-1.1]) \cdot \mathbf{i} + (-5.6 \cdot 2.4 - [-1.4] \cdot 3.2) \cdot \mathbf{j} + ([-1.4] \cdot [-1.1] - 4.2 \cdot 2.4) \cdot \mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = \underline{7 \cdot \mathbf{i} - 9 \cdot \mathbf{j} - 9 \cdot \mathbf{k}}$$

Just as we did before, we can calculate the magnitudes of the two vectors and then determine the angle between them using Equation (1.9). The only difference now is that the magnitude of the cross product is not as easy to determine. We must calculate it like we calculate the magnitudes of all vectors:

$$A=\sqrt{(-1.4)^2+4.2^2+(-5.6)^2}=7.1$$

$$B=\sqrt{2.4^2+1.1^2+3.2^2}=4.1$$

$$|AxB|=\sqrt{7^2+(-9)^2+(-9)^2}=15$$

Now that we have all of the magnitudes involved, we can use Equation (1.9):

$$|AxB|=A\cdot B\cdot\sin\theta$$

$$15=(7.1)\cdot(4.1)\cdot\sin\theta$$

$$\theta=\sin^{-1}\left(\frac{15}{(7.1)\cdot(4.1)}\right)=\underline{31^{\circ}}$$

ON YOUR OWN

1.13 Vector **A** is defined as 3.2 feet at 45.0°, and vector **B** is defined as 1.1 feet at 70.1°. What is the cross product? Give your answer in unit vector notation.

1.14 Given the following vectors, calculate the cross product and the angle between them:

$$\mathbf{A} = -7.1\cdot\mathbf{i} + 4.2\cdot\mathbf{j}$$
$$\mathbf{B} = 3.4\cdot\mathbf{i} - 4.1\cdot\mathbf{j}$$

<u>The Physical Significance of the Cross Product</u>

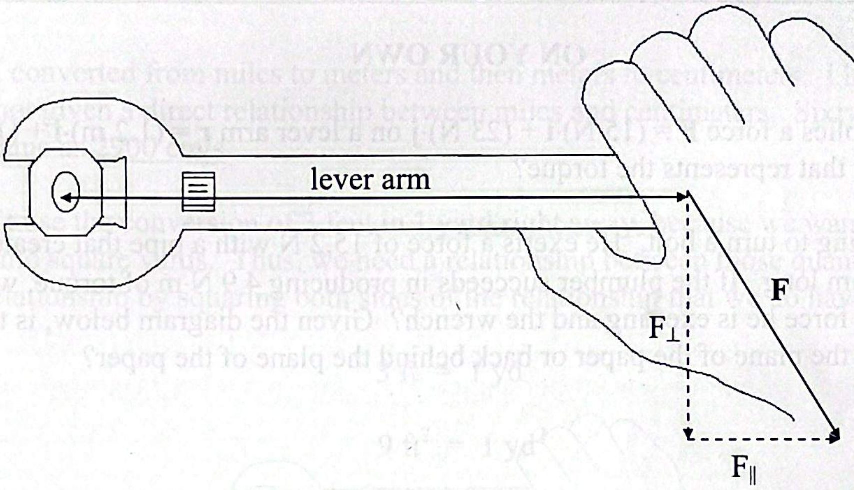
As was the case with the dot product, there is physical significance to the cross product. We will apply the cross product in at least three different areas of physics, but for right now, I will concentrate on only one: the concept of **torque**. As you learned in your first-year physics course, when we apply a force some distance away from an axis of rotation, the result is a torque, which can cause rotational motion. In your first-year physics course, you learned the equation used to calculate torque.

$$\tau = F_{\perp}\cdot r \qquad (1.12)$$

Where "τ" represents torque, "r" represents the magnitude of the vector drawn from the axis of rotation to the point at which the force is applied, and "$F_\perp$" represents the component of the force which is perpendicular to that vector. Figure 1.11 illustrates the concept of torque.

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FIGURE 1.11
Torque



Now remember what torque is. It is the counterpart of force when one is considering *rotational motion*. Remember, force causes acceleration in a straight line. Torque causes rotational acceleration. In the figure, the wrench is going to turn the screw. To do that, it will have to give the screw rotational acceleration so that it starts to turn in a circle. Torque is the impetus which will cause that rotational acceleration.

Notice from the figure that only a portion of the force used can generate torque. Any component of the force that is parallel to the vector defined from the axis of rotation to the point at which the force is applied (the **lever arm**) is lost. Thus, the magnitude of the torque is given by the magnitude of the lever arm times the component of the force which is perpendicular to the lever arm.

Well, since the cross product takes the magnitude of a vector and multiplies it by the magnitude of a second vector's component which is perpendicular to the first vector, torque can be calculated using the cross product.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \qquad (1.13)$$

There are two things to note about this equation. First, torque is a vector. Remember, the cross product results in a vector. That's why the "τ" is bold. Second, remember that the cross product

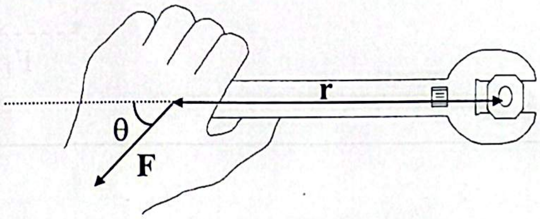
is NOT commutative. Thus, the order of the vectors *is* important, so one must take the vector of the lever arm (**r**) and cross it with the vector representing the force (**F**), *in that order*.

Since you already know how to do cross products, calculating the torque is pretty easy. Thus, I will not give you any examples of it. However, you should perform the following "on your own" problems to make sure you understand how to use the cross product to calculate torque.

ON YOUR OWN

1.15 A person applies a force **F** = (15 N)·**i** + (23 N)·**j** on a lever arm **r** = (1.2 m)·**i** + (1.1 m)·**j**. What is the vector that represents the torque?

1.16 A man is trying to turn a bolt. He exerts a force of 15.2 N with a pipe that creates a lever arm which is 0.36 m long. If the plumber succeeds in producing 4.9 N·m of torque, what is the angle between the force he is exerting and the wrench? Given the diagram below, is the torque pointing up above the plane of the paper or back behind the plane of the paper?



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Summing Up

This module contained some concepts which were review for you and others which were new. That will be the case with most of the modules in this course. In each module, I will review some of the highlights of your first-year physics course and then go deeper into each subject. In addition, completely new concepts will be brought in from time to time. Thus, if there is something that really baffles you in this course, you should review your first-year course to see if it is explained there.

ANSWERS TO THE ON YOUR OWN PROBLEMS

1.1 To solve this problem, we simply have to convert miles to centimeters and hours to seconds. It doesn't matter that they are both a part of our original unit. As long as we use the factor-label method, everything will work out:

$$\frac{65\ \text{miles}}{1\ \text{hour}} \times \frac{1609\ \text{m}}{1\ \text{mile}} \times \frac{1\text{cm}}{0.01\text{m}} \times \frac{1\ \text{hour}}{3600\ \text{s}} = 2900\ \frac{\text{cm}}{\text{s}}$$

Notice that I converted from miles to meters and then meters to centimeters. I had to do that, since I was not given a direct relationship between miles and centimeters. Sixty-five miles per hour is the same as 2900 cm/s.

1.2 We can't use the conversion of 3 feet in 1 yard right away, because we want to convert from square feet into square yards. Thus, we need a relationship between those quantities. We can get such a relationship by squaring both sides of the relationship that we do have:

$$3\ \text{ft} = 1\ \text{yd}$$

$$9\ \text{ft}^2 = 1\ \text{yd}^2$$

Now we can do the conversion:

$$\frac{1600\ \text{ft}^2}{1} \times \frac{1\ \text{yd}^2}{9\ \text{ft}^2} = 180\ \text{yd}^2$$

The house has an area of 180 square yards. If your answer was 177.8, go back and review significant figures from your first-year course.

1.3 To convert from Joules ($\frac{\text{kg}\cdot\text{m}^2}{\text{sec}^2}$) to ergs ($\frac{\text{g}\cdot\text{cm}^2}{\text{sec}^2}$), I need to convert kg into g and m^2 into cm^2. That's not bad. We know that 1 kg = 1,000 g. We also know:

$$1\ \text{cm} = 0.01\ \text{m}$$

To get the relationship between cm^2 and m^2, we just square both sides:

$$1\ \text{cm}^2 = 0.0001\ \text{m}^2$$

This leads us to:

$$\frac{151\ \text{kg}\cdot\text{m}^2}{1\ \text{sec}^2} \times \frac{1{,}000\,\text{g}}{1\ \text{kg}} \times \frac{1\ \text{cm}^2}{0.0001\ \text{m}^2} = 1.51\times 10^9\ \frac{\text{g}\cdot\text{cm}^2}{\text{sec}^2}$$

1.4 In this problem, we are given the x- and y-components of a vector and are asked to calculate its magnitude and direction. Getting the magnitude is not too bad:

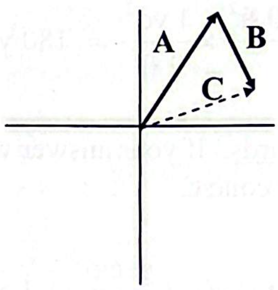
$$\text{Magnitude} = \sqrt{V_x^{\,2} + V_y^{\,2}} = \sqrt{(23\,\frac{\text{m}}{\text{sec}})^2 + (11\,\frac{\text{m}}{\text{sec}})^2} = 25\,\frac{\text{m}}{\text{sec}}$$

To get the angle, we start with this equation:

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{11\frac{\text{m}}{\text{sec}}}{23\frac{\text{m}}{\text{sec}}}\right) = 26^{\circ}$$

We aren't necessarily finished yet, however. We have to determine which region of the Cartesian coordinate system that the vector is in. Since both its components are positive, this tells us that the vector is to the right and above the origin, which means that the vector is in region I. According to our rules, we don't need to do anything to the result of the equation when the vector is in region I, so 26° is the proper angle. Thus, the vector has magnitude of 25 m/sec and direction of 26°.

1.5 In this problem, we are asked to add two-dimensional vectors together. Before we do this mathematically, let's do it graphically:



As we learned before, the dotted arrow (vector C) gives us the sum.

The first step in adding vectors mathematically is to break both vectors down into their components:

$$A_x = (3.1\,\frac{\text{m}}{\text{sec}})\cdot\cos(60.0^{\circ}) = 1.6\,\frac{\text{m}}{\text{sec}}$$

$$A_y = (3.1\,\frac{\text{m}}{\text{sec}})\cdot\sin(60.0^{\circ}) = 2.7\,\frac{\text{m}}{\text{sec}}$$

$$B_x = (1.4\ \frac{m}{sec})\cdot\cos(290.0^\circ) = 0.48\ \frac{m}{sec}$$

$$B_y = (1.4\ \frac{m}{sec})\cdot\sin(290.0^\circ) = -1.3\ \frac{m}{sec}$$

Now that we have the individual components, we can add them together.

$$C_x = A_x + B_x = 1.6\ \frac{m}{sec} + 0.48\ \frac{m}{sec} = 2.1\ \frac{m}{sec}$$

$$C_y = A_y + B_y = 2.7\ \frac{m}{sec} + -1.3\ \frac{m}{sec} = 1.4\ \frac{m}{sec}$$

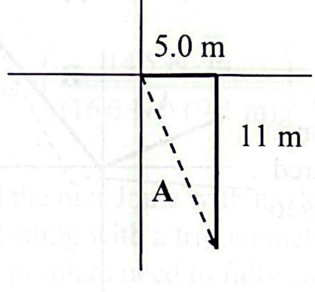
Now that we have the components to our answer, we can get the magnitude and direction of the sum.

$$\text{Magnitude} = \sqrt{C_x^{\ 2} + C_y^{\ 2}} = \sqrt{(2.1\ \frac{m}{sec})^2 + (1.4\ \frac{m}{sec})^2} = 2.5\ \frac{m}{sec}$$

$$\theta = \tan^{-1}(\frac{C_y}{C_x}) = \tan^{-1}(\frac{1.4\frac{m}{sec}}{2.1\frac{m}{sec}}) = 34^\circ$$

Since the x- and y-components are both positive, the vector is in the first region of the Cartesian coordinate plane. This is consistent with the graphical answer we drew to begin with, and it means that we do not need to do anything to the result the equation. Thus, the sum of vectors **A** and **B** has a magnitude of 2.5 m/sec at a direction of 34°.

1.6 To draw the vector, we simply need to realize that the number multiplying **i** is the x-component and the number multiplying **j** is the y-component.



To get the magnitude and direction, we just use the formulas in Figure 1.2.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(5.0\,\text{m})^2 + (-11\,\text{m})^2} = 12\ \text{m}$$

$$\theta = \tan^{-1}\left(\frac{-11\ \text{m}}{5.0\ \text{m}}\right) = -66^\circ$$

Now remember, the angle may not be defined correctly at this point. Based on the components, we see that this vector is in region IV of the Cartesian coordinate plane. Thus, we must add 360.0° to it. Thus, the answer is 12 m at a direction of 294°.

1.7 To add vectors, we add the x-components and y-components. In unit vector notation, the formula is given by Equation (1.2):

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} = (5\ \text{m} + -11\ \text{m})\mathbf{i} + (-11\ \text{m} + 22\text{m})\mathbf{j} = \underline{(-6\ \text{m})\mathbf{i} + (11\ \text{m})\mathbf{j}}$$

The difference is given by Equation (1.3):

$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} = (5\ \text{m} - -11\text{m})\mathbf{i} + (-11\ \text{m} - 22\ \text{m})\mathbf{j} = \underline{(16\ \text{m})\mathbf{i} - (33\ \text{m})\mathbf{j}}$$

1.8 In unit vector notation, scalar multiplication is given by Equation (1.2):

$$k\mathbf{A} = (kA_x)\mathbf{i} + (kA_y)\mathbf{j}$$

$$6\mathbf{C} = (6\text{x}2.0\ \text{m/sec})\mathbf{i} + (6\text{x}3.0\ \text{m/sec})\mathbf{j} = \underline{(12\ \text{m/sec})\mathbf{i} + (18\ \text{m/sec})\mathbf{j}}$$

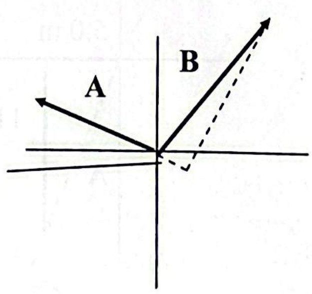
1.9 Using Equation (1.5):

$$\mathbf{A} \bullet \mathbf{B} = A_x \cdot B_x + A_y \cdot B_y$$

$$\mathbf{A} \bullet \mathbf{B} = (-2.3\ \text{m})\cdot(1.2\ \text{m}) + (-1.2\ \text{m})\cdot(4.3\ \text{m}) = -8.0\ \text{m}^2$$

The dot product, then, equals $\underline{-8.0\ \text{m}^2}$. What does the minus sign mean? Well, remember, the dot product is the product of the magnitude of **A** and the magnitude of the component of **B** parallel to **A**. If that product is negative, it means the component of **B** parallel to **A** is negative. That means the component of **B** parallel to **A** points opposite of **A**. Draw the two vectors to see what I mean:

Component of **B** parallel to **A** points in the opposite direction compared to **A**. Thus, compared to **A**, it is negative. This leads to a negative dot product.



1.10 This problem requires you to understand that the dot product **A•B** is the magnitude of **A** times the magnitude of the component of **B** which lies parallel to **A**. You are given those two things, so you can compute the dot product:

$$\mathbf{A} \bullet \mathbf{B} = (\text{magnitude of } \mathbf{A}) \cdot (\text{magnitude of the component of } \mathbf{B} \text{ parallel to } \mathbf{A})$$

$$\mathbf{A} \bullet \mathbf{B} = (15.1 \text{ m}) \cdot (1.2 \text{ m}) = 18 \text{ m}^2$$

Now that we have the dot product, we can solve Equation (1.6) for the magnitude of **B**.

$$\mathbf{A} \bullet \mathbf{B} = A \cdot B \cdot \cos\theta$$

$$18 \text{ m}^2 = (15.1 \text{ m}) \cdot B \cdot \cos(61.0^\circ)$$

$$B = \frac{18 \text{ m}^2}{(15.1 \cancel{\text{m}}) \cdot \cos(61.0^\circ)} = \underline{2.5 \text{ m}}$$

1.11 This is a straightforward application of Equation (1.8):

$$W = \mathbf{F} \bullet \mathbf{x} = (5.6 \text{ N}) \cdot (1.5 \text{ m}) + (3.4 \text{ N}) \cdot (2.3 \text{ m}) = 16.2 \text{ N}\cdot\text{m} = \underline{16.2 \text{ J}}$$

Remember from your first-year course that a N·m is the standard unit for energy or work, and that unit is called the **Joule** in honor of James Prescott Joule.

1.12 Remember, work is the dot product of **F** and **x**. Thus,

$$W = F \cdot x \cdot \cos\theta$$

In this problem, we have all of the variables except θ, so we can solve for it:

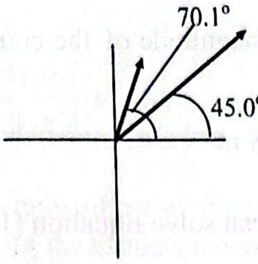
$$W = F \cdot x \cdot \cos\theta$$

$$14.5 \text{ J} = (16.6 \text{ N}) \cdot (9.2 \text{ m}) \cdot \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{14.5 \cancel{\text{N}} \cdot \cancel{\text{m}}}{(16.6 \cancel{\text{N}}) \cdot (9.2 \cancel{\text{m}})}\right) = \underline{85^\circ}$$

Notice that in the last line, I replaced the unit Joule with its definition, N·m, to show that the units cancel out. Remember, when dealing with a trigonometric function, the argument should have no units. Thus, any units in the problem need to fully cancel, as they do above.

1.13 To calculate the magnitude of the cross product, we just use Equation (1.9). To do that, however, we need the angle between the vectors:



This angle is easy, it's just 70.1° - 45.0° = 25.1°. Now we can use Equation (1.9):

$$|\mathbf{A} \times \mathbf{B}| = A \cdot B \cdot \sin\theta = (3.2\text{ feet}) \cdot (1.1\text{ feet}) \cdot \sin(25.1^\circ) = 1.5\text{ feet}^2$$

To determine direction, we use the right hand rule. We point the fingers of our right hand in the direction of the first vector, then we curl along the arc of the 25.1° angle in between the vectors. When we do that, our thumb points up out of the page. Thus, the vector is in the positive **k** direction. That tells us what we need to know for unit vector notation. Thus, the vector is $\underline{(1.5\text{ feet}^2) \cdot \mathbf{k}}$.

1.14 Notice that the vectors are two-dimensional and have only **i** and **j** unit vectors. Thus, we can use the simpler version of the cross product formula, Equation (1.10):

$$\mathbf{A} \times \mathbf{B} = (A_x \cdot B_y - A_y \cdot B_x) \cdot \mathbf{k}$$

$$\mathbf{A} \times \mathbf{B} = [(-7.1) \cdot (-4.1) - 4.2 \cdot 3.4] \cdot \mathbf{k} = \underline{15 \cdot \mathbf{k}}$$

That's the cross product. What about the angle? Well, we just need the magnitudes of all three vectors and Equation (1.9). The magnitude of the cross product is easy; it's just 15. What about **A** and **B**?

$$A = \sqrt{(-7.1)^2 + (4.2)^2} = 8.2$$

$$B = \sqrt{(3.4)^2 + (-4.1)^2} = 5.3$$

Now we can use Equation (1.9):

$$|\mathbf{A} \times \mathbf{B}| = A \cdot B \cdot \sin\theta$$

$$15 = (8.2) \cdot (5.3) \cdot \sin\theta$$

$$\theta = \sin^{-1}\left(\frac{15}{(8.2) \cdot (5.3)}\right) = \underline{2.0 \times 10^{1}\ ^{\circ}}$$

Notice that I had to put the answer in scientific notation. The rules indicate that I must have 2 significant figures. The number 20 has only 1 significant figure. Thus, I had to make that zero significant by putting the whole number in scientific notation.

1.15 This is an easy one. We are given the vectors for **r** and **F**, so we can just use Equation (1.13). Since these are two-dimensional vectors in the **i**/**j** plane, we can use the simplified cross product formula, Equation (1.10).

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = (r_x \cdot F_y - r_y \cdot F_x) \cdot \mathbf{k} = [(1.2\text{ m}) \cdot (23\text{ N}) - (1.1\text{ m}) \cdot (15\text{ N})] \cdot \mathbf{k} = \underline{(11\text{ N} \cdot \text{m}) \cdot \mathbf{k}}$$

1.16 In this problem, we are dealing only with magnitudes. Since we want to figure out the angle between the vectors, we are going to use Equation (1.9):

$$|\mathbf{A} \times \mathbf{B}| = A \cdot B \cdot \sin\theta$$

$$4.9\,\text{N} \cdot \text{m} = (0.36\,\text{m}) \cdot (15.2\,\text{N}) \cdot \sin\theta$$

$$\theta = \sin^{-1}\left(\frac{4.9\,\cancel{\text{N}} \cdot \cancel{\text{m}}}{(0.36\,\cancel{\text{m}}) \cdot (15.2\,\cancel{\text{N}})}\right) = \underline{64^{\circ}}$$

To determine the direction, we use the right hand rule. Point the fingers of your right hand from the nut to the hand. That is along the lever arm. Now, curl your fingers towards the force, along the arc of the angle between the lever arm and the force. Your thumb should be pointed up away from the paper. Thus, <u>the torque points above the plane of the paper</u>.

REVIEW QUESTIONS FOR MODULE #1

1. What is the conversion relationship between m^2 and km^2?

2. Two vectors are written below. Draw them on a Cartesian coordinate plane so that they each start at the origin.

A: 55 miles/hour at 45 degrees **B**: 20 miles/hour at 210 degrees

3. A position vector, **P**, is given as 36 miles at 15 degrees. What are the magnitude and angle of -2·**P**?

4. In which region of the Cartesian coordinate system do you find the vector **A** = 1.2·**i** - 3.4·**j**?

5. You are given the vector **S** = -3.4·**i** - 4.5·**j** and must determine the angle at which it is pointing. You use the fact that $\tan\theta = S_y/S_x$ to get the angle. However, what do you have to do to the result of that equation in order to report the angle so that it is defined properly?

6. The dot product **A•B** is equal to 34.2 m^2. What is the value of **B•A**?

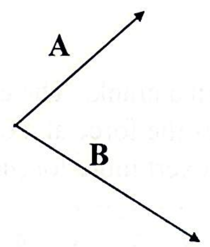
7. Fill in the blank:

The dot product takes the magnitude of one vector and multiplies it by the magnitude of the second vector's component which is _____________ to the first vector.

8. Suppose a person applies a force to an object, but the object travels in a direction that is perpendicular to the force. How much work is being done?

9. The cross product **A** x **B** is equal to (45 m^2/sec^2)·**k**. What is the cross product **B** x **A**?

10. Given the diagram below, determine the direction of the vector **A** x **B**. Is the vector going back behind the plane of the paper or coming out above the plane of the paper?



PRACTICE PROBLEMS FOR MODULE #1

1. Convert an acceleration of 845 feet/hour^2 into meters/sec^2. (1 m = 3.281 feet)

2. A Newton is a $\frac{kg \cdot m}{sec^2}$. Convert 15.1 Newtons into $\frac{g \cdot cm}{minute^2}$.

3 The velocity vector of an object is **v** = (13 m/sec)·**i** - (11 m/sec)·**j**. What are the magnitude and direction of the velocity vector?

4. Vector **A** has a magnitude of 2.2 m/sec at an angle of 35.0 degrees, and vector **B** has a magnitude of 3.4 m/sec at an angle of 120.0 degrees. Graphically add these two vectors together, and then compute the magnitude and direction.

5. Given the following vectors:

A = (1.5 miles)·**i** + (7.1 miles)·**j**
B = (1.8 miles)·**i** - (2.2 miles)·**j**

What is **A** + **B**? What is **A** - **B**? Give your answers in unit vector notation.

6. Given the following vector:

C = (12.0 m)·**i** + (13.0 m)·**j**

What is -5·**C** in unit vector notation?

7. An object undergoes a displacement **x** = (-5.2 m)·**i** + (1.1 m)·**j** while being acted upon by a constant force **F** = (1.5 N)·**i** - (4.3 N)·**j**. What is the work done?

8. A person applies a force of 18 Newtons to an object as the object travels 3.4 meters. The angle between the force and the displacement is 35.1 degrees. How much work was done?

9. A construction worker is using a crowbar to pry a rock out of the ground. He applies a force **F** = (-55 N)·**i** - (34 N)·**j** + (43 N)·**k** at a lever arm **r** = (0.23 m)·**i** + (0.25 m)·**j** + (0.22 m)·**k**. What is the vector that represents the torque?

10. A person applies a force in order to turn a crank. The crank is 0.50 meters long, and he applies a force of 15 Newtons. If he applies the force at a 65 degree angle relative to the crank, what is the torque? How could the person exert more torque without increasing the force or changing the lever arm?